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On the Reliability of Regression Models

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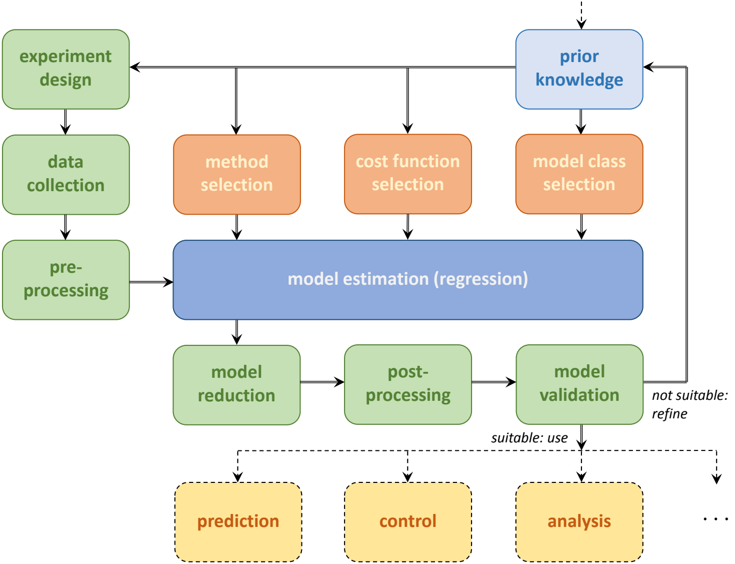
Constructing and Applying Regression Models

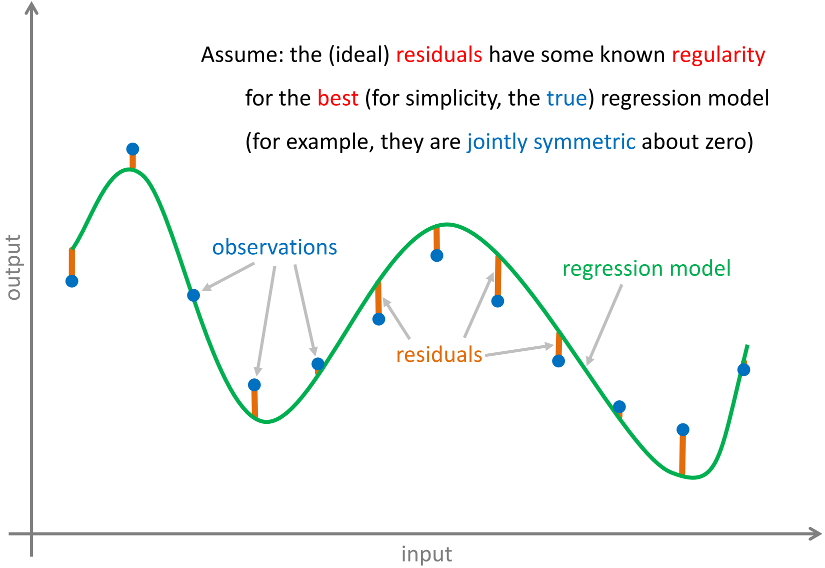


Regularity Assumption

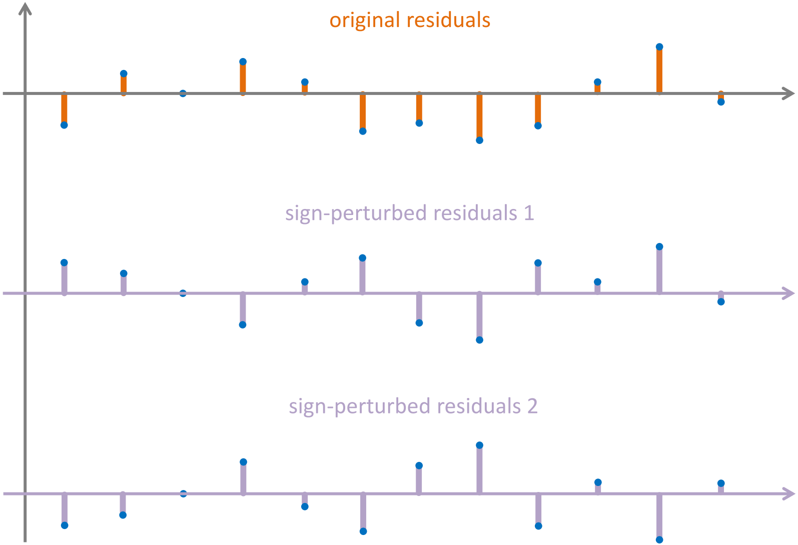
Assume: the (ideal) residuals have some known regularity

for the best (for simplicity, the true) regression model

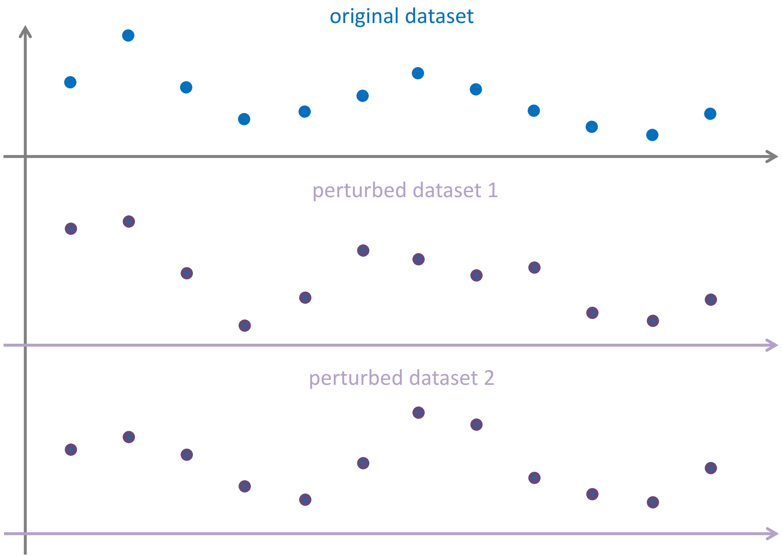
(for example, they are jointly symmetric about zero)



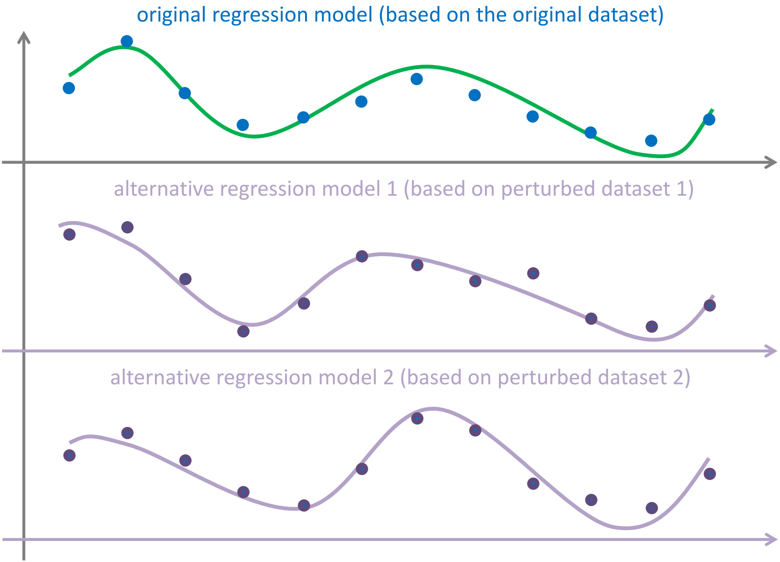
Perturbed Residuals



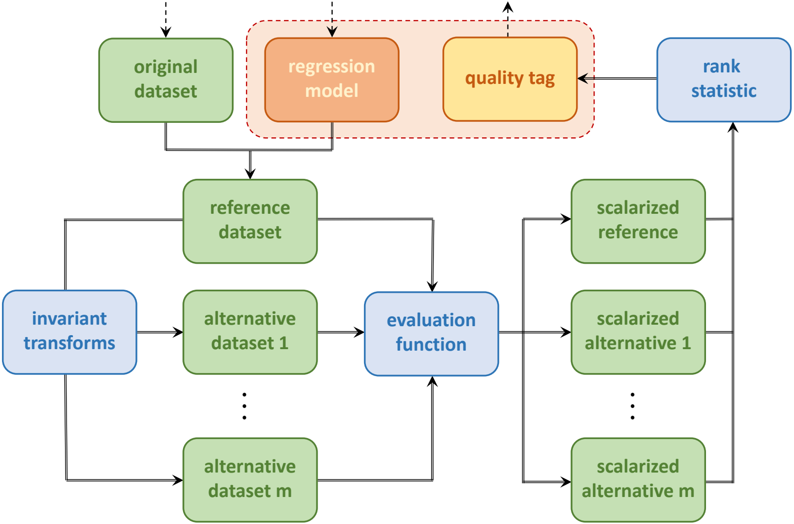
Perturbed Datasets



Alternative Regression Models



Quality Tags



Dynamical Systems

Many (discrete-time) dynamical system models in science and engineering can be formalized as controlled Markov chains.

Dynamical System (Markov)

$$x_t \triangleq f(x_{t-1}, u_t, w_t)$$

where

- t — time (discrete)
- x_t — output (state)
- u_t — input (external)
- w_t — noise (innovation)
- f — transition function

Point Estimation

Consider the parametric estimation problem of the system

$$x_t \triangleq f_{\theta^*}(x_{t-1}, u_t, w_t),$$

parametrized with $\theta^* \in \Theta \subseteq \mathbb{R}^d$

Given: a finite sample, $\mathcal{Z}$, of outputs $\{x_t\}$ and inputs $\{u_t\}$

Point Estimate (Parametric)

$$\widehat{\theta}_{\mathcal{Z}} \triangleq \operatorname*{arg\,min}_{\theta \in \Theta} \mathcal{V}(\theta \mid \mathcal{Z})$$

where $\mathcal{V}$ is a criterion function.

Confidence Regions

In practice often some quality tag is needed to judge the estimate.

Safety, stability, or quality requirements? $\Rightarrow$ confidence regions

Confidence Region (Level μ)

$$\mathbb{P}\big(\theta^* \in \widehat{\Theta}_{\mathcal{Z},\mu}\big) \geq \mu$$

for some $\mu \in (0,1)$, where θ^* is the "true" parameter, $\widehat{\Theta}_{\mathcal{Z},\mu} \subseteq \Theta$.

Typically the level sets of the (scaled) limiting distribution is used.

Issues: only approximately correct for finite samples,
requires the existence of a (known) limiting distribution.

Main Objectives

- We aim at building confidence regions for dynamical systems.
- With non-asymptotic guarantees ("finite sample" method).
- Which are distribution-free: namely, do not make strong statistical assumption on the innovations of the process.
- They should be built around specific point estimates.
- The Sign-Perturbed Sums (SPS) method is presented.
- Its main assumption is that the noise terms are symmetric.
- Under which it can even provide exact confidence sets.
- Main examples: linear regression, general linear dynamical systems (including closed-loop systems), volatility models.

Linear Regression

Consider a standard linear regression problem:

Linear Regression

$$x_t \triangleq \varphi_t^{\mathrm{T}} \theta^* + w_t$$

where

x_t — output (for time $t = 1, \dots, n$)

φ_t — regressor (deterministic, d dimensional)

w_t — noise (zero mean, uncorrelated)

σ^2 — variance of the noise (homoscedastic)

θ^* — true parameter (deterministic, d dimensional)

$\Phi_n = [\varphi_1, \dots, \varphi_n]^{\mathrm{T}}$ — skinny and full rank

Least Squares

Given: a sample, $\mathcal{Z}$, of size n of outputs $\{x_t\}$ and regressors $\{\varphi_t\}$

A classical approach is the least squares criterion, namely

$$\mathcal{V}(\theta \mid \mathcal{Z}) \triangleq \frac{1}{2}\sum_{t=1}^{n}(x_t - \varphi_t^{\mathrm{T}}\theta)^2.$$

The least squares estimate (LSE) can be found by solving

Normal Equation

$$\nabla_\theta \mathcal{V}(\hat{\theta}_n \mid \mathcal{Z}) = \sum_{t=1}^{n}\varphi_t(x_t - \varphi_t^{\mathrm{T}}\hat{\theta}_n) = 0$$

Asymptotic Normality

LSE can be explicitly formulated as

$$\hat{\theta}_n = \Bigg(\sum_{t=1}^{n} \varphi_t \varphi_t^{\mathrm{T}}\Bigg)^{-1} \Bigg(\sum_{t=1}^{n} \varphi_t x_t\Bigg).$$

LSE is asymptotically normal

Limiting Distribution

$$\sqrt{n}\,(\hat{\theta}_n - \theta^*) \xrightarrow{d} \mathcal{N}(0, \sigma^2 R^{-1}) \quad \text{as} \quad n \to \infty$$

where R is the limit of $R_n = \frac{1}{n} \sum_{t=1}^{n} \varphi_t \varphi_t^{\mathrm{T}}$ as $n \to \infty$ (if exists).

Confidence Ellipsoids

The standard confidence region construction is then

Confidence Ellipsoid

$$\widetilde{\Theta}_{n,\mu} \triangleq \left\{ \theta \in \mathbb{R}^d \,:\, (\theta - \hat{\theta}_n)^{\mathrm{T}} R_n (\theta - \hat{\theta}_n) \,\leq\, \frac{\mu\, \hat{\sigma}_n^2}{n} \right\}$$

where $\mathbb{P}(\theta^* \in \widetilde{\Theta}_{n,\mu}) \approx F_{\chi^2(d)}(\mu)$, where $F_{\chi^2(d)}$ is the CDF of $\chi^2(d)$,

$$\hat{\sigma}_n^2 \triangleq \frac{1}{n-d} \sum_{t=1}^{n} (x_t - \varphi_t^{\mathrm{T}} \hat{\theta}_n)^2,$$

is an estimate of σ^2 based on the sample.

Reference and Sign-Perturbed Sums

Let us introduce a reference sum and $m-1$ sign-perturbed sums.

Reference Sum

$$S_0(\theta) \triangleq R_n^{-\frac{1}{2}} \sum_{t=1}^{n} \varphi_t (x_t - \varphi_t^{\mathrm{T}} \theta)$$

Sign-Perturbed Sums

$$S_i(\theta) \triangleq R_n^{-\frac{1}{2}} \sum_{t=1}^{n} \varphi_t \, \alpha_{i,t} (x_t - \varphi_t^{\mathrm{T}} \theta)$$

for $i = 1, \dots, m-1$, where $\alpha_{i,t}$ $(t = 1, \dots, n)$ are i.i.d. random signs, that is $\alpha_{i,t} = \pm 1$ with probability $1/2$ each (Rademacher).

Intuitive Idea: Distributional Invariance

Assume $\{w_t\}$ are independent and each w_t is symmetric about zero.

Observe that, if $\theta = \theta^*$, we have $(i = 1, \dots, m-1)$

Distributional Invariance

$$S_0(\theta^*) = R_n^{-\frac{1}{2}} \sum_{t=1}^{n} \varphi_t w_t$$

$$S_i(\theta^*) = R_n^{-\frac{1}{2}} \sum_{t=1}^{n} \varphi_t \, \alpha_{i,t} w_t$$

Consider the ordering $\|S_{(0)}(\theta^*)\|^2 \prec \cdots \prec \|S_{(m-1)}(\theta^*)\|^2$

Note: relation "$\prec$" is the canonical "$<$" with random tie-breaking

All orderings are equally probable! (they are conditionally i.i.d.)

Intuitive Idea: Reference Dominance

What if $\theta \neq \theta^*$?

In fact, the reference paraboloid $\|S_0(\theta)\|^2$ increases faster than $\{\|S_i(\theta)\|^2\}$, thus will eventually dominate the ordering.

Intuitively, for "large enough" $\|\tilde{\theta}\|$, where $\tilde{\theta} \triangleq \theta^* - \theta$

Eventual Dominance of the Reference Paraboloid

$$\Big\| \sum_{t=1}^{n} \varphi_t \varphi_t^{\mathrm{T}} \tilde{\theta} + \sum_{t=1}^{n} \varphi_t w_t \Big\|_{R_n^{-1}}^2 > \Big\| \sum_{t=1}^{n} \pm \varphi_t \varphi_t^{\mathrm{T}} \tilde{\theta} + \sum_{t=1}^{n} \pm \varphi_t w_t \Big\|_{R_n^{-1}}^2$$

with "high probability" (for simplicity $\pm$ is used instead of $\{\alpha_{i,t}\}$).

Non-Asymptotic Confidence Regions

The rank of $\|S_0(\theta)\|^2$ in the ordering of $\{\|S_i(\theta)\|^2\}$ w.r.t. $\prec$ is

$$\mathcal{R}(\theta) = 1 + \sum_{i=1}^{m-1} \mathbb{I}(\|S_i(\theta)\|^2 \prec \|S_0(\theta)\|^2),$$

where $\mathbb{I}(\cdot)$ is an indicator function.

Sign-Perturbed Sums (SPS) Confidence Regions

$$\widehat{\Theta}_n \triangleq \left\{ \theta \in \mathbb{R}^d \,:\, \mathcal{R}(\,\theta\,) \leq m - q \right\}$$

where $m > q > 0$ are user-chosen integers (design parameters).

Exact Confidence

(A1) $\{w_t\}$ is a sequence of independent random variables.

Each w_t has a symmetric probability distribution about zero.

(A2) The outer product of regressors is invertible, $\det(R_n) \neq 0$.

Exact Confidence of SPS

$$\mathbb{P}\big(\, \theta^* \in \widehat{\Theta}_n \,\big) \,=\, 1 - \frac{q}{m}$$

for finite samples. Parameters m and q are under our control.

Note that $\|S_0(\hat{\theta}_n)\|^2 = 0$, thus $\hat{\theta}_n \in \widehat{\Theta}_n$, assuming it is non-empty.

Star Convexity

Set $\mathcal{X} \subseteq \mathbb{R}^d$ is star convex if there is a star center $c \in \mathbb{R}^d$ with

$$\forall x \in \mathcal{X}, \forall \beta \in [0,1] : \beta x + (1-\beta) c \in \mathcal{X}.$$

Star Convexity of SPS

$\widehat{\Theta}_n$ is star convex with the LSE, $\hat{\theta}_n$, as a star center

Hint $\widehat{\Theta}_n$ is the union and intersection of ellipsoids containing LSE.

Strong Consistency

(A1) independence, symmetricity: $\{w_t\}$ are independent, symmetric

(A2) invertibility: $R_n \triangleq \frac{1}{n}\sum_{t=1}^{n} \varphi_t \varphi_t^{\mathrm{T}}$ is invertible

(A3) regressor growth rate: $\sum_{t=1}^{\infty} \|\varphi_t\|^4 / t^2 < \infty$

(A4) noise moment growth rate: $\sum_{t=1}^{\infty} \left(\mathbb{E}[w_t^2]\right)^2 / t^2 < \infty$

(A5) Cesàro summability: $\lim_{n\to\infty} R_n = R$, which is positive definite

Strong Consistency of SPS

$$\mathbb{P}\left(\bigcup_{k=1}^{\infty} \bigcap_{n=k}^{\infty} \left\{ \widehat{\Theta}_n \subseteq B_\varepsilon(\theta^*) \right\} \right) = 1,$$

where $B_\varepsilon(\theta^*) \triangleq \{\, \theta \in \mathbb{R}^d : \|\theta - \theta^*\| \leq \varepsilon \,\}$ is a norm ball.

Ellipsoidal Outer Approximation

The reference paraboloid can be rewritten as

$$\|S_0(\theta)\|^2 = (\theta - \hat{\theta}_n)^{\mathrm{T}} R_n (\theta - \hat{\theta}_n).$$

From which an alternative description of the confidence region is

$$\widehat{\Theta}_n \subseteq \left\{ \theta \in \mathbb{R}^d \,:\, (\theta - \hat{\theta}_n)^{\mathrm{T}} R_n (\theta - \hat{\theta}_n) \leq r(\theta) \right\},$$

where $r(\theta)$ is the qth largest value of $\{\|S_i(\theta)\|^2\}_{i \neq 0}$.

Ellipsoidal Outer Approximation

$$\widehat{\Theta}_n \subseteq \left\{ \theta \in \mathbb{R}^d \,:\, (\theta - \hat{\theta}_n)^{\mathrm{T}} R_n (\theta - \hat{\theta}_n) \leq r^* \right\}$$

The question is of course how to find such an r^* efficiently.

Quadratically Constrained Quadratic Program

$\max\{\|S_i(\theta)\|^2 : \|S_0(\theta)\|^2 \leq \|S_i(\theta)\|^2\}$ can be obtained by

$$\begin{array}{ll} \text{maximize} & \|z\|^2 \\ \text{subject to} & z^{\mathrm{T}} A_i z + 2 z^{\mathrm{T}} b_i + c_i \leq 0 \end{array}$$

$$\begin{aligned} A_i &\triangleq I - R_n^{-\frac{1}{2}} Q_i R_n^{-1} Q_i R_n^{-\frac{1}{2}\mathrm{T}}, \\ b_i &\triangleq R_n^{-\frac{1}{2}} Q_i R_n^{-1} (\psi_i - Q_i \hat{\theta}_n), \\ c_i &\triangleq -\psi_i^{\mathrm{T}} R_n^{-1} \psi_i + 2 \hat{\theta}_n^{\mathrm{T}} Q_i R_n^{-1} \psi_i - \hat{\theta}_n^{\mathrm{T}} Q_i R_n^{-1} Q_i \hat{\theta}_n. \end{aligned}$$

$$Q_i \triangleq \sum_{t=1}^{n} \alpha_{i,t} \varphi_t \varphi_t^{\mathrm{T}}, \qquad \psi_i \triangleq \sum_{t=1}^{n} \alpha_{i,t} \varphi_t x_t.$$

Semi-Definite Program

Problem: the previous QCQP is not convex.

Fortunately, strong duality holds and its dual can be written as:

Dual Problem

$$\begin{aligned} \text{minimize} \quad & \gamma \\ \text{subject to} \quad & \lambda \geq 0 \\ & \begin{bmatrix} -I + \lambda A_i & \lambda b_i \\ \lambda b_i^{\mathrm{T}} & \lambda c_i + \gamma \end{bmatrix} \succeq 0 \end{aligned}$$

where "$\succeq 0$" denotes that a matrix is positive semidefinite.

Radius r^* can then be found by solving $m - 1$ such convex problems, obtaining $\{\gamma_i^*\}$, and defining r^* the qth largest one.

Simulation Experiment

Finite Impulse Response (FIR) System (2nd order)

$$x_t = 0.7\, u_{t-1} + 0.3\, u_{t-2} + w_t$$

where $\{w_t\}$ are i.i.d. zero mean Laplacian, with variance 0.1.

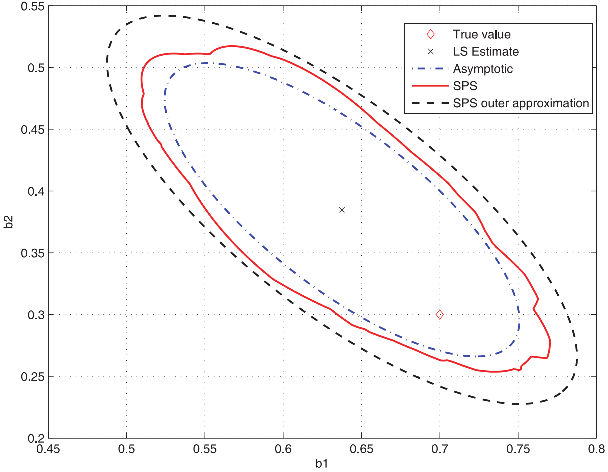
The input signal $\{u_t\}$ is given by the autoregression

$$u_t = 0.75\, u_{t-1} + v_t,$$

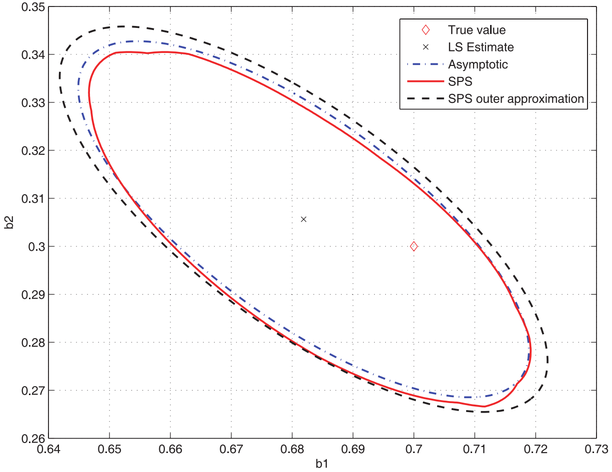
where $\{v_t\}$ is a sequence of i.i.d. standard normal variables.

Confidence regions (level 95 %) of SPS, its outer approximation and the standard asymptotic ellipsoids are compared.

95% Confidence Regions, $n = 25$, $m = 100$, $q = 5$



95% Confidence Regions, $n = 400$, $m = 100$, $q = 5$



Linear Dynamical Systems

Consider systems written using (rational) transfer functions:

General Linear Systems

$$x_t \triangleq G(z^{-1};\theta^*)\,u_t + H(z^{-1};\theta^*)\,w_t$$

(A1) The true system is in the model class, the orders are known.

(A2) The transfer function $H(z^{-1};\theta^*)$ has a (stable) inverse, as well as $G(0;\theta^*) = 0$ and $H(0;\theta^*) = 1$.

(A3) Noises $\{w_t\}$ are independent and symmetrically distributed.

(A4) Inputs $\{u_t\}$ are observed and independent of $\{w_t\}$.

(A5) Initialization: for all $t \leq 0$, we have $x_t = w_t = u_t = 0$.

Prediction Error Estimate

Prediction Error or Residual (for parameter θ)

$$\widehat{\varepsilon}_t(\theta) \triangleq H^{-1}(z^{-1};\theta)\left(x_t - G(z^{-1};\theta)\,u_t\right)$$

Note that $\widehat{\varepsilon}_t(\theta^*) = w_t$, hence, it is accurate for $\theta = \theta^*$.

Prediction Error Estimate (for model class Θ)

$$\hat{\theta}_{\mathrm{PEM}} \triangleq \operatorname*{arg\,min}_{\theta\in\Theta} \sum_{t=1}^{n} \widehat{\varepsilon}_t^{\,2}(\theta)$$

In general, there is no closed-form solution for PEM.

Prediction Error Equation

The PEM estimate can be found, e.g., by using the equation

PEM Equation

$$\nabla_\theta \mathcal{V}(\hat{\theta}_{\mathrm{PEM}} \,|\, \mathcal{Z}) \,=\, \sum_{t=1}^{n} \psi_t(\hat{\theta}_{\mathrm{PEM}})\,\widehat{\varepsilon}_t(\hat{\theta}_{\mathrm{PEM}}) \,=\, 0$$

where $\psi_t(\theta)$ is the negative gradient of the prediction error,

$$\psi_t(\theta) \,\triangleq\, -\nabla_\theta\,\widehat{\varepsilon}_t(\theta).$$

These gradients can be directly calculated in terms of the defining polynomials of the rational transfer functions G and H.

Perturbed Samples

Perturbed Output Trajectories

$$\bar{x}_t(\theta, \alpha_i) \triangleq G(z^{-1}; \theta)\, u_t + H(z^{-1}; \theta)\, (\alpha_{i,t}\, \widehat{\varepsilon}_t(\theta))$$

where $\{\alpha_{i,t}\}$ are random signs, as previously.

Recall that $\psi_t(\theta)$ is a linear filtered version of $\{x_t\}$ and $\{u_t\}$,

$$\psi_t(\theta) = W_0(z^{-1}; \theta)\, x_t + W_1(z^{-1}; \theta)\, u_t,$$

where W_0 and W_1 are vector-valued, and $\psi_t(\theta) \in \mathbb{R}^d$.

Perturbed (Negative) Gradients

$$\bar{\psi}_t(\theta, \alpha_i) \triangleq W_0(z^{-1}; \theta)\, \bar{x}_t(\theta, \alpha_i) + W_1(z^{-1}; \theta)\, u_t$$

Sign-Perturbed Sums for General Linear Systems

Reference and Sign-Perturbed Sums

$$S_0(\theta) \triangleq \Psi_n^{-\frac{1}{2}}(\theta) \sum_{t=1}^{n} \psi_t(\theta)\, \widehat{\varepsilon}_t(\theta)$$

$$S_i(\theta) \triangleq \bar{\Psi}_n^{-\frac{1}{2}}(\theta, \alpha_i) \sum_{t=1}^{n} \bar{\psi}_t(\theta, \alpha_i)\, \alpha_{i,t}\, \widehat{\varepsilon}_t(\theta)$$

Reference and Sign-Perturbed Covariances

$$\Psi_n(\theta) \triangleq \frac{1}{n} \sum_{t=1}^{n} \psi_t(\theta) \psi_t^{\mathrm{T}}(\theta)$$

$$\bar{\Psi}_n(\theta, \alpha_i) \triangleq \frac{1}{n} \sum_{t=1}^{n} \bar{\psi}_t(\theta, \alpha_i) \bar{\psi}_t^{\mathrm{T}}(\theta, \alpha_i)$$

Non-Asymptotic Confidence Regions

$\mathcal{R}(\theta)$ is again the rank of $\|S_0(\theta)\|^2$ among $\{\|S_i(\theta)\|^2\}$ w.r.t. $\prec$

SPS Confidence Regions for General Linear Systems

$$\widehat{\Theta}_n \triangleq \left\{ \theta \in \mathbb{R}^d \,:\, \mathcal{R}(\,\theta\,) \leq m - q \,\right\}$$

where $m > q > 0$ are user-chosen (integer) parameters.

We have $S_0(\hat{\theta}_{\mathrm{PEM}}) = 0$, thus, $\hat{\theta}_{\mathrm{PEM}} \in \widehat{\Theta}_n$, if it is non-empty.

Exact Confidence of SPS for General Linear Systems

$$\mathbb{P}\big(\,\theta^* \in \widehat{\Theta}_n\,\big) \;=\; 1 - \frac{q}{m}$$

Simulation Experiment

Autoregressive Moving Average: ARMA(1,1)

$$x_t + a^* x_{t-1} = w_t + c^* w_{t-1}$$

where $\theta^* = (a^*, c^*)$ and $\{w_t\}$ are i.i.d. standard normal.

The inverse filter of $C(z^{-1}; \theta)\, w_t = w_t + c\, w_{t-1}$ is

$$C^{-1}(z^{-1}; \theta) = \sum_{k=0}^{\infty} (-1)^k \, c^k \, z^{-k}$$

Can be used to define the prediction errors for $\theta = (a, c)$

$$\widehat{\varepsilon}_t(\theta) = C^{-1}(z^{-1}; \theta)\, (x_t + a\, x_{t-1}).$$

Simulation Experiment

Perturbed Output Trajectories

$$\bar{x}_t(\theta, \alpha_i) \;=\; -a\,\bar{x}_{t-1}(\theta, \alpha_i) + \alpha_{i,t}\,\widehat{\varepsilon}_t(\theta) + c\,\alpha_{i,t-1}\widehat{\varepsilon}_{t-1}(\theta)$$

for $1 \leq i \leq m$ and $1 \leq t \leq n$, where $\{\alpha_{i,t}\}$ are random signs.

Perturbed (Negative) Gradients

$$\bar{\psi}_t(\theta, \alpha_i) \;=\; \begin{bmatrix} -C^{-1}(z^{-1};\theta)\,\bar{x}_t(\theta, \alpha_i) \\ C^{-1}(z^{-1};\theta)\,\alpha_{i,t}\,\widehat{\varepsilon}_t(\theta) \end{bmatrix}$$

which can be used to define the sign-perturbed sums.

99% Confidence Regions, $n = 500$, $m = 100$, $q = 1$

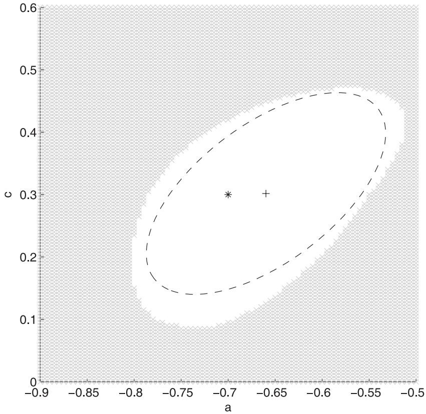


Figure: "$\times$": SPS (compl.); "$*$": θ^*; "$+$": PEM; "- -": asymp. ellipsoid

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Closed-Loop General Linear System

Dynamical System: General Linear

$$Y_t \triangleq G(z^{-1}; \theta^*)\, U_t + H(z^{-1}; \theta^*)\, N_t$$

t : (discrete) time, Y_t : output, U_t : input, N_t : noise, R_t : reference,
G, H : transfer functions, z^{-1} : backward shift, θ^* : true parameter.

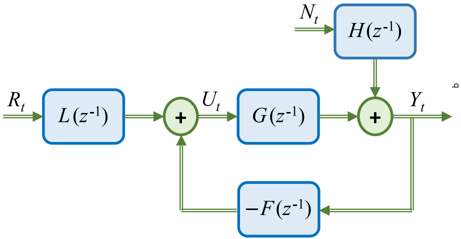
Controller: Closed-Loop with Reference Signal

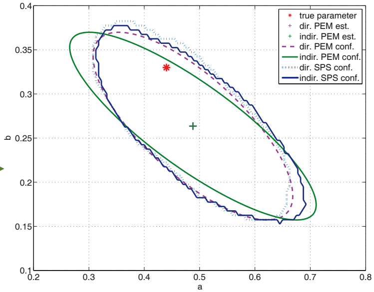
$$U_t \triangleq L(z^{-1}; \eta^*)\, R_t - F(z^{-1}; \eta^*)\, Y_t$$

L, F : transfer functions parametrized independently of G, H.

Closed-Loop Prediction Error Methods (PEMs)

- SPS can be extended to closed-loop linear systems, to the direct, indirect and joint input-output approach of PEM.
- The constructed confidence regions remain exact.





Extension to GARCH Processes

- Formally, a GARCH(p, q) process, $\{X_t\}$, is defined by

$$X_t \triangleq \sigma_t \, \varepsilon_t,$$

$$\sigma_t^2 \triangleq \omega^* + \sum_{i=1}^{p} \alpha_i^* X_{t-i}^2 + \sum_{j=1}^{q} \beta_j^* \sigma_{t-j}^2,$$

where $\{\varepsilon_t\}$ is i.i.d., $\mathbb{E}[\varepsilon_t] = 0$ and $\mathbb{E}[\varepsilon_t^2] = 1$

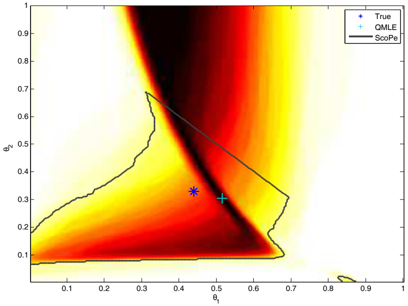
- $\theta^* \triangleq (\omega^*, \alpha_1^*, \dots, \alpha_p^*, \beta_1^*, \dots, \beta_q^*)$ are nonnegative, $\omega^* > 0$
- Quasi-maximum likelihood methods typically optimize

$$\ell_n(\theta) \triangleq \frac{1}{n} \sum_{t=1}^{n} \left[\log \widehat{\sigma}_t^2(\theta) + \frac{X_t^2}{\widehat{\sigma}_t^2(\theta)} \right],$$

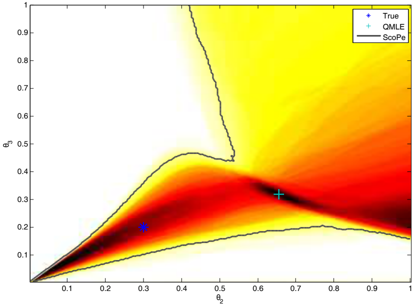
where $\widehat{\sigma}_t^2(\theta)$ is the estimated variance process based on θ.

Extension to GARCH Processes

- The constructed regions are exact and work well on real data. (Compound retuns on Nasdaq 100, S&P 500 and FTSE 100.)



Logistic noise, 90% confidence set for stationary GARCH(1, 1)



Lagrangian noise, 90% confidence set for stationary GARCH(1, 1)

Further Extensions

- Instrumental variable methods
- General correlation methods
- Least absolute deviation based methods
- Regularized regression
- Robustness analysis and robustifycation techniques
- Undermodelling detection

- Approximations via interval analysis
- Input perturbation / arbitrary noises
- Robust model predictive control
- Distributed confidence set computation

Summary

- A finite sample estimation framework was presented.
- Prime example: the SPS (Sign-Perturbed Sums) method.
- It builds confidence regions around the least squares estimate.
- Only mild statistical assumptions are needed, e.g., symmetry.
- Not needed: stationarity, moments, particular distributions.
- For (rational) probabilities, exact confidence sets can be built.
- SPS is strongly consistent, i.e., the confidence regions almost surely shrink around the true parameter (for lin.reg.).
- SPS is star convex with the LSE as a center (for lin.reg.).
- Efficient ellipsoidal outer approximation exists (for lin.reg.).
- The framework has many extensions, can handle closed-loop LTI systems, GARCH processes, and LAD and IV methods.

Thank you for your attention!

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